is considerably smaller than if the controlled wheel were not present. For example, with an eccentricity of 0.1, a reduction of 50% in the amplitude of the natural libration can be expected.

The controlled rotor gravity gradient stabilization system offers an inexpensive open-loop approach to providing three-axis attitude control. Since attitude sensors and complex torquing devices can be eliminated, this new stabilization concept should find many applications.


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# On Murphy's Stability Criterion


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### Introduction

MURPHY[1] has pointed out the reason for the failure of the stability criterion of Kelley and McShane to reduce to the classical stability condition $s > 1$ ($s$ = stability factor) while discussing the effect of overturning moment coefficient only. The same criterion is derived here afresh, using the direct method of Liapunov, and it is shown how the method reduces to that of Murphy's.

### Construction of Liapunov Function

The equations of motion of a projectile[2] are written in the matrix form

$$\dot{X} = AX \tag{1}$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$x_1, x_2$ are complex values of the linear and angular yaw velocity, respectively; $a_{11}, a_{12}, a_{21}, a_{22}$ are complex force and moment coefficients. Overhead dot $(\cdot)$ denotes differentiation with respect to arc length.

The characteristic equation

$$|A - \lambda I| = 0 \tag{2}$$

of the matrix $A$ has roots with negative real parts if[3]

$$\sigma_1 \leqq 0 \tag{3}$$

$$\sigma_1\sigma_2\sigma_4 + \sigma_1{}^2\sigma_3 - \sigma_4{}^2 > 0 \tag{4}$$

where $\sigma_i$'s are the stability parameters, and $\sigma_1$ denotes the real part of $-(a_{11}+a_{22})$, $\sigma_2$ denotes the imaginary part of $-(a_{11}+a_{22})$, $\sigma_3$ denotes the real part of $(a_{11}a_{22}-a_{21}a_{12})$, $\sigma_4$ denotes the imaginary part of $(a_{11}a_{22}-a_{21}a_{12})$.

Let $\lambda_1$, $\lambda_2$ be the roots (complex and distinct) of Eq. (2). Now, by a theorem of Poincaré,[4] the system (1) is transformed into the canonical form

$$\dot{z}_1 = \lambda_1 z_1 \qquad \dot{z}_2 = \lambda_2 z_2 \tag{5}$$

We choose the following positive definite Hermitian form as a Liapunov function for the system (5) that is

$$L(z) = z_1\bar{z}_1 + z_2\bar{z}_2 \tag{6}$$

Its derivative

$$\dot{L}(z) = (Rl\lambda_1)z_1\bar{z}_1 + (Rl\lambda_2)z_2\bar{z}_2 \tag{7}$$

is, obviously, negative definite, subject to the conditions of Eqs. (3) and (4). Hence, the stability of the system (1) is concluded.

### Murphy's Condition

Using conditions (3) and (4) and the relation

$$4(\sigma_1\sigma_2\sigma_4 + \sigma_1{}^2\sigma_3 - \sigma_4{}^2) + (\sigma_1{}^2)^2 + (\sigma_1\sigma_2 - 2\sigma_4)^2 > 0 \tag{8}$$

we derive the auxiliary condition

$$\sigma_1{}^2 + \sigma_2{}^2 + 4\sigma_3 > 0 \tag{9}$$

In the case when $\sigma_1 = 0$, the condition (9) becomes

$$\sigma_2{}^2 + 4\sigma_3 > 0 \tag{10}$$

which yields, if only the moment coefficient is considered in the notation of Murphy

$$\bar{v}^2 - 4M > 0 \tag{11}$$

that is

$$s > 1 (s = \bar{v}^2/4M) \tag{12}$$

Further, when spin $v$ is zero, we have

$$M < 0 \tag{13}$$

that is

$$\phi_M < 0 \tag{14}$$

and the conclusions drawn by Murphy follow immediately.

### Conclusions

As a consequence of the present investigation, it is found that, though the major stability conditions [Eqs. (3) and (4)] are essentially the same as were given by Kelley and McShane, the auxiliary condition (9), on which Murphy's discussion of the effect of overturning moment coefficient on stability would be argued, is implicit in their derivation and, hence, the reason for its failure to reduce to the gyroscopic condition $s > 1$.

Moreover, the method suggested is, also, free from the type of objection raised by Laitone.[5]


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# Total Pressure Recovery for a Dump Combustor

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### Nomenclature

$A$ = area
$k$ = ratio of specific heats
$M$ = Mach number
$p, p_T$ = pressure and total pressure, respectively
$R$ = universal gas constant
$T, T_T$ = temperature and total temperature, respectively
$w$ = flow rate
$\Delta T/T = (T_{T_4} - 1)/T_{T_3}$
$\eta_p$ = combustor pressure efficiency

*Subscripts*
2 = diffuser exit
3 = combustor entrance
4 = combustor exit
5 = nozzle throat

### Introduction

THE over-all total pressure recovery for a combustor can be obtained by combining the total pressure recovery across the combustor entrance and the total pressure recovery across the combustor. These two pressure recoveries are generally determined by separate methods. A method has been developed from one-dimensional gasdynamics which predicts the over-all total pressure recovery for a combustor within 2% of experimental results. The method is applicable for combustors in which the incoming flow undergoes a rather sudden expansion. Such combustors are referred to as dump combustors and are used in after-burners and ramjet engines. A schematic of a dump combustor with station notation is shown in Fig. 1.

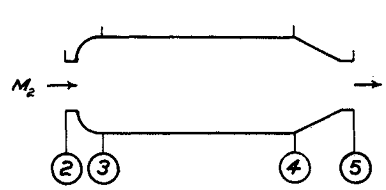


Fig. 1 Schematic of dump combustor.

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### Development of Total Pressure Recovery Relation

There are two main sources of total pressure loss in a dump combustor. The first is viscous losses associated with the sudden expansion of the flow across the dump or combustor entrance. The second is fluid acceleration as a result of the heat addition.

The pressure recovery across the dump $p_{T_3}/p_{T_2}$ will be determined by analyzing the dump as a sudden enlargement, as is done in pipeline flow. References 1 and 2 show good agreement with experimental results using this approach.

The momentum and continuity equations across a sudden enlargement are

$$p_3A_3(1+kM_3^2) = p_2A_2(1+kM_2^2)+p_2(A_3-A_2) \quad (1)$$

$$p_3A_3M_3[1+(k-1)M_3^2/2]^{1/2} = p_2A_2M_2[1+(k-1)M_2^2/2]^{1/2} = w[RT_{T_2}/k]^{1/2} \quad (2)$$

The perfect gas relations were assumed in writing Eqs. (1) and (2). Combining Eqs. (1) and (2), and using the isentropic relations for total to static properties, one can determine the dump total pressure recovery $p_{T_3}/p_{T_2}$. The relationship for $p_{T_3}/p_{T_2}$ is presented functionally by Eq. (3) and graphically in Fig. 2.

$$p_{T_3}/p_{T_2} = f(M_2, A_3/A_2, k) \quad (3)$$

The total pressure losses across the combustor are due to heat addition and friction. The combined effect of heat addition and friction can be analyzed for a compressible fluid using the methods outlined in Ref. 3. The frictional losses in a flow-through combustor are generally small and will be neglected in this analysis. The combustor pressure recovery $p_{T_4}/p_{T_3}$ will be predicted from the Rayleigh line relationships, which are

$$p_{T_4}/p_{T_3} = \{(1+kM_3^2)/(1+kM_4^2)\} \times \{[1+(k-1)M_4^2/2]/[1+(k-1)M_3^2/2]\}^{k/(k-1)} \quad (4)$$

$$\Delta T/T = (T_{T_4}/T_{T_3}) - 1 = \{[M_4^2(1+kM_3^2)^2]/[M_3^2(1+kM_4^2)^2]\} \times \{[1+(k-1)M_4^2/2]/[1+(k-1)M_3^2/2]\} - 1 \quad (5)$$

It can be seen from Eqs. (4) and (5) for a given $M_4$ that, as the temperature rise term $\Delta T/T$ increases, $M_3$ and $p_{T_4}/p_{T_3}$ decrease. The relationship between $p_{T_4}/p_{T_3}$ and $\Delta T/T$ is presented in Fig. 3. From Fig. 2, it can be seen that, as $M_3$ decreases, the dump pressure recovery $p_{T_3}/p_{T_2}$ increases. Therefore, opposing effects exist across the entire combustor with the addition of heat—as $\Delta T/T$ increases, $p_{T_4}/p_{T_3}$ decreases, and $p_{T_3}/p_{T_2}$ increases, and vice versa.

These opposing effects have a compensating effect on the over-all combustor pressure recovery and result in $p_{T_4}/p_{T_2}$ being fairly constant for a wide range of heat additions and/or fuel air ratios. The limiting or maximum value from $p_{T_4}/p_{T_2}$ is the

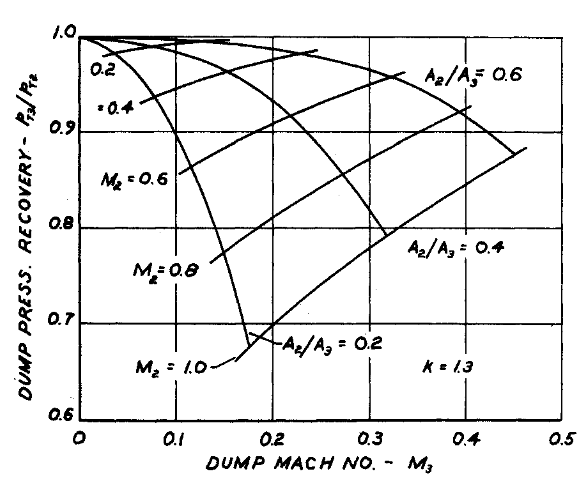


Fig. 2 Dump total pressure recovery.